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Cambridge Assessment
International Education



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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1 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular.

[3]

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(b) Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the value of k .

[2]

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2 The cubic equation $x^3 + 2x^2 + 3x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^2 + 1, \beta^2 + 1, \gamma^2 + 1$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

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(b) Find the value of $(\alpha^2 + 1)^2 + (\beta^2 + 1)^2 + (\gamma^2 + 1)^2$. [2]

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(c) Find the value of $(\alpha^2 + 1)^3 + (\beta^2 + 1)^3 + (\gamma^2 + 1)^3$. [2]

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The triangle DEF in the x - y plane is transformed by $\mathbf{M}$ onto triangle PQR .

- (c) Given that the area of triangle PQR is 35 cm^2 , find the area of triangle DEF . [2]

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4 (a) Prove by mathematical induction that, for all positive integers n ,

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1). \quad [5]$$

[illegible]



The sum S_n is defined by $S_n = \sum_{r=1}^n r^4$.

(b) Using the identity

$$(2r+1)^5 - (2r-1)^5 \equiv 160r^4 + 80r^2 + 2,$$

show that $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$. [6]

[illegible]

(c) Find the value of $\lim_{n \rightarrow \infty} (n^{-5} S_n)$. [2]

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- 5 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2 . [5]

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The plane Π_1 contains l_1 and is parallel to l_2 .

(b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

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(c) The point $(1, 1, 1)$ lies on the plane Π_γ .

It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0, 0, 2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

Obtain an equation of Π_2 in the form $ax+by+cz=d$. [3]

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6 The curve C has equation $y = \frac{x+1}{x^2+3}$.

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

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(b) Find the coordinates of any stationary points on C . [4]

[illegible]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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(d) Sketch $y^2 = \frac{x+1}{x^2+3}$, stating the coordinates of the stationary points and the intersections with the axes. [4]





7 The curve C has polar equation $r^2 = \sin 2\theta \cos \theta$, for $0 \leq \theta \leq \pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find a Cartesian equation for C . [3]

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(c) Find the total area enclosed by C .

[4]

[illegible]



(d) Find the greatest distance of a point on C from the pole.

[6]

[illegible]



Additional page

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